EFFECTIVE STRENGTH PARAMETERS OF COMPOSITES IN COUPLED PHYSICOMECHANICAL FIELDS

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The development of a modern technology, operated under complex conditions of interacting physical factors, has stimulated the creation of a theory of coupled (e.g., electromagnetic) fields in elastic bodies, particularly composites. In the mechanics of microinhomogeneous media with a random structure highly effective methods have been developed for calculating the linear effective properties of a medium in various physicomechanical fields from the properties of the components of the medium and a comparison with experiments has been made [1-6]. As a rule, an estimate is made of the average values of various fields in components that depend linearly on the random fields in the medium; this substantially simplifies the solution of the indicated classical problem. The prediction of the strength properties of a medium is complicated in a fundamental manner by the essentially nonlinear dependence of the strength laws on the local field strength. For example, the simplest method of estimating the effective parameters of the mechanical strength of composites is the rule of mixtures [7], which is invariant under the form and orientation of the inclusions. More exact methods are based on calculations of the average field strengths in composites, but as shown below a study of only the first moments of the field strength [6] in the components leads to qualitative errors in the estimate of the effective strength parameters of composites.

In this paper we propose a method of constructing the strength surface of a composite in static coupled physicomechanical fields with allowance for the coupling of the fields, arbitrary anisotropy of the physicomechanical properties, and shape and orientation of the filler inclusions; the methods of estimating the one-point first [1] and second [2] moments of the coupled fields in the components of the medium are used.

1. General Relations. We consider a macroregion z with a characteristic function Z at each point of which a local equation

$$\sigma^\alpha(x) = L^{\alpha\beta}(x)\varepsilon^\beta(x), \quad \nabla\sigma^\alpha(x) = 0 \quad (\alpha,\ \beta = 1, \ldots, N), \tag{1.1}$$

associates the tensors of the thermodynamic forces ε^β and fluxes σ^α; tensors of the mechanical stresses, electromagnetic field induction, heat flow, mass flow, etc., can be used as σ^α; ε^β can denote strain tensors, field tensors, etc. The fields ε^β are assumed to be potential fields ($\varepsilon^\beta = \nabla u^\beta$), ∇ is the operator of the symmetrized gradient and the gradient when acting on a tensor of the first and zero rank, respectively. The tensors of the physicomechanical properties $L^{\alpha\beta}$ of the second, third, or fourth ranks are independent of ε^γ ($\gamma = 1,\ldots,N$) and satisfy the Onsager conditions and the energy limitations.

We assume that at every point of the medium there is a tensor-polynomial strength criterion, which is invariant under transformations of the coordinate system,

$$\sum_\alpha \left(\Pi^{1\alpha}(x)\sigma^\alpha(x) + \Pi^{2\alpha}(x)\sigma^\alpha(x)\otimes\sigma^\alpha(x)\right) = 1, \tag{1.2}$$

and is analogous to the Malmeister strength criterion [8]; tensors $\Pi^{1\alpha}$ and $\Pi^{2\alpha}$ are of the same ranks as tensors σ^α and $\sigma^\alpha\otimes\sigma^\alpha$ ($\otimes$ is the sign for the tensor product).

Suppose that the region z consists of a matrix v_0 with characteristic function V with homogeneous coefficients $L_0^{\alpha\beta}, \Pi_0^{1\alpha}, \Pi_0^{2\alpha}$ and the random set $X = (V_k, z_k, \omega_k)$ of ellipsoidal inclusions with characteristic functions V_k, and centers x_k, forming a Poisson set, semiaxes a_k $(a_k^1 \geqslant a_k^2 \geqslant a_k^3)$, a set of Euler angles ω_k and homogeneous coefficients $L^{\alpha\beta}(x) = L_0^{\alpha\beta} + L_1^{\alpha\beta}(x) \equiv L_0^{\alpha\beta} + L_1^{\alpha\beta(k)}$, $\Pi^{1\alpha}(x) = \Pi_0^{1\alpha} + \Pi_1^{1\alpha}(x) \equiv \Pi_0^{1\alpha} + \Pi_1^{1\alpha(k)}$, $\Pi^{2\alpha}(x) = \Pi_0^{2\alpha} + \Pi_1^{2\alpha(k)}$ $(x \in v_k)$.

Along with the tensor form we also use the equivalent matrix notation of with a standard transformation rule. We introduce the vectors $\sigma \equiv (\sigma^\alpha)$, $\varepsilon \equiv (\varepsilon^\alpha)$, $u \equiv (u^\alpha)$, $\Pi_0^1 = (\Pi_0^{1\alpha})$, $\Pi_1^1 = (\Pi_1^{1\alpha})$

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and the matrices $L_0 = (L_0^{\alpha\beta})$, $L_1 = (L_1^{\alpha\beta})$, $\Pi_0^2 = (\Pi_0^{2\alpha})$, $\Pi_1^2 = (\Pi_1^{2\alpha})$. All of the random fields considered here are assumed to be ergodic, statistically uniform random fields and the averaging over the ensemble X can thus be replaced by averaging over the macrovolume. We introduce the notation

$$\langle(\cdot)\rangle = (\bar{z})^{-1}\int(\cdot)\,\boldsymbol{Z}(x)\,dx,\quad \langle(\cdot)\rangle_k = (\bar{v}_k)^{-1}\int(\cdot)\,V_k(x)\,dx,\quad k = 0, 1, \ldots$$

Here and below the integration is carried out over the entire region of z; the bar above it indicates its measure, $\bar{z}$ = mes z.

To find the tensor L^* of effective properties of the equation of the macrostate $\langle\sigma\rangle = L^*\langle\varepsilon\rangle$ we average the local equations of state (1.1) over the macrovolume, whereupon

$$L^* = L_0 + R^*, \tag{1.3}$$

where R^* is determined by the mean of the polarization tensor $\langle VL_1\varepsilon\rangle \equiv R^*\langle\varepsilon\rangle$, $V = \Sigma V_k$ (summation over k = 1, 2,...).

An alternative, although equivalent, method can be proposed for introducing the effective tensor L^* on the basis of the energy considerations:

$$\langle\varepsilon(x)\sigma(x)\rangle = \langle\varepsilon\rangle L^*\langle\varepsilon\rangle. \tag{1.4}$$

The equivalence stems from the general condition (analogous to the Hill condition [10])

$$\langle\varepsilon(x)\sigma(x)\rangle = \langle\varepsilon\rangle\langle\sigma\rangle, \tag{1.5}$$

which is not related to any specific equation of state and was obtained on the basis of only the equilibrium equation $\nabla\sigma = 0$ and the Cauchy equation $\varepsilon = \nabla u^c$:

$$\langle\varepsilon(x)\,\sigma(x)\rangle = (\bar{z})^{-1}\int\langle\varepsilon(x)\,\sigma(x)\rangle\,dx = (\bar{z})^{-1}\left\langle\int\nabla\,[u(x)\,\sigma(x)]\,dx\right\rangle = (\bar{z})^{-1}\oint\langle u(x)\,\sigma_n(x)\rangle\,ds.$$

Here integration is carried out over the region z and its boundary ∂z with the normal n; $\sigma_n = \sigma n$. Assuming that the boundary condition are determined in terms of the thermodynamic fluxes $\sigma_n(x) = \langle\sigma(x)\rangle n(x)$, $x \in \partial z$, we obtain

$$\langle\varepsilon(x)\,\sigma(x)\rangle = (\bar{z})^{-1}\oint\langle u(x)\,n(x)\rangle\,ds\,\langle\sigma(x)\rangle = (\bar{z})^{-1}\int\langle\varepsilon(x)\rangle\,ds\,\langle\sigma(x)\rangle,$$

and from this we have (1.5), which can be proven for boundary conditions in potentials u.

For a fixed uniform field $\langle\varepsilon\rangle$ we vary the local property tensor $L(x) \to L(x) + \delta L(x)$, whereupon from (1.5) at a fixed $\langle\varepsilon\rangle$ we find

$$\langle\delta L^*\langle\varepsilon\rangle\rangle\langle\varepsilon\rangle = \langle(\delta L\varepsilon)\varepsilon + 2\delta\varepsilon(L\varepsilon)\rangle. \tag{1.6}$$

We assume that the property tensors and their variation are uniform within the limits of the component v_k (k = 0,1,...):

$$L(x) = \sum_{k=0} L^{(k)}V_k(x),\quad \delta L(x) = \sum_{k=0}\delta L^{(k)}V_k(x).$$

Then in Eq. (1.6) transformed by the scheme of [2] the tensor δL can be taken out in front of the sign of averaging over the volume v_k of the component, which makes it possible to recast (1.6) as

$$\langle\varepsilon\otimes\varepsilon\rangle_k = \bar{v}_k^{-1}\partial L^*/\partial L^{(k)}\,(\langle\varepsilon\rangle\otimes\langle\varepsilon\rangle)\quad (k = 0, 1, \ldots), \tag{1.7}$$

i.e., the tensors of the second moments of the field strength $\langle\varepsilon\otimes\varepsilon\rangle_k$, averaged over the volume of component k, are determined uniquely from the functional dependence $L^* = L^*(L^{(k)})$. Writing (1.4) in terms of the thermodynamic fluxes σ: $(M^*\langle\sigma\rangle)\langle\sigma\rangle = \langle(M\sigma)\sigma\rangle$, we have an expression for the second moment of the tensor of thermodynamic fluxes, analogous to (1.7):

$$\langle\sigma\otimes\sigma\rangle_k = \bar{v}_k^{-1}\partial M^*/\partial M^{(k)}\,(\langle\varepsilon\rangle\otimes\langle\varepsilon\rangle). \tag{1.8}$$

Here we have introduced the reciprocal tensors $M^{(k)} \equiv (L^{(k)})^{-1}$, $M^* \equiv (L^*)^{-1}$.

We go on to evaluate tensor R^* (1.3). To abbreviate the manipulations we assume that L_0 is a cellular-diagonal matrix $L_0 \equiv \mathrm{diag}\,[L_0^{11},\ldots,L_0^{NN}]$, i.e., the effects of the coupling of the fluxes are not taken into account in the matrix. In the general case of matrix L_0 the transition from it to the cellular-diagonal matrix by introducing modified variables was considered in [11, 12]. Then Eq. (1.1) can be rewritten as

$$\nabla L_0\nabla u = -\nabla L_1\nabla u. \tag{1.9}$$

Since analytical representations or numerical methods for constructing the fundamental solution $G^{\alpha\alpha}$ exist for most uncoupled physicomechanical fields described by the equations $\nabla L_0^{\alpha\alpha} \times \nabla u^{\alpha} = 0$, we determine a cellular-diagonal block matrix of the fundamental solutions $G = (G^{\alpha\beta})$, $G^{\alpha\beta} = 0$ for $\alpha \neq \beta$. Then with the assumptions described in [1] Eq. (1.9) reduces to the integral

$$\varepsilon(x) = \langle\varepsilon\rangle - \int U(x-y)[L_1(y)\varepsilon(y)V(y) - \langle L_1\varepsilon V\rangle]\,dy \tag{1.10}$$

($U = \nabla\nabla G$). The convergence of the integral in (1.10) was studied in [1]. We evaluate R^* from (1.10) within the framework of the multiparticle effective field method (EMF).

We introduce $\varphi(v_m|x_1, \ldots, x_n)$, arbitrary densities of the probability of finding the m-th inclusion in region v_m for fixed inclusions $v_1,\ldots,v_n$ with centers $x_1,\ldots,x_n$. We average (1.10) on the set X for fixed values of v_1; v_1, v_2, etc., by means of various densities of the distribution $\varphi(v_m|x_1, \ldots, x_n)$. We obtain an infinite system of coupled equations, which we average of the volume of the i-th inclusion ($i = 1,\ldots,n$) and have

$$\begin{gathered}\langle\varepsilon\,|\,x_1,\ldots x_n\rangle_i - \sum_{j=1}^{n}\bar{v}_i^{-1}\int\int U(x-y)V_i(x)V_j(y)\,\times\\ \times\,\langle L_1(y)\varepsilon(y)\,|\,y;\,x_1,\ldots,x_n\rangle\,dy\,dx = \langle\varepsilon\rangle + \bar{v}_i^{-1}\int\int U(x-y)V_i(x)\times\\ \times[\langle V(y)L_1(y)\varepsilon(y)\,|\,y;\,x_1,\ldots,x_n\rangle - \langle L_1\varepsilon V\rangle]\,dy\,dx,\end{gathered} \tag{1.11}$$

where $\langle(\cdot)|y;\, x_1,\ldots,x_n\rangle$ is the condition for averaging over the ensemble X when $y, x_1,\ldots,x_n$ are inclusions and $y \neq x_1,\ldots,x_n$.

We denote the right sides of the equations by $\langle\hat{\varepsilon}(x)_{1,\ldots,n}\rangle_i$, and $\bar{\varepsilon}_i = \bar{\varepsilon}(x_i)$ is the field in which the i-th inclusion is located, then ($j = 1,\ldots,n$).

$$\bar{\varepsilon}_i(x) = \hat{\varepsilon}(x)_{1,\ldots,n} + \sum_{j\neq i}^{n}\int U(x-y)V_j(y)L_1(y)\varepsilon(y)\,dy. \tag{1.12}$$

2. The Effective Field Method. For the approximation solution of Eqs. (1.11), (1.12) we adopt the EMF hypothesis about the uniformity of the field $\bar{\varepsilon}(x_i)$ and closure at sufficiently large n,

$$(\langle\hat{\varepsilon}(x)_{1,\ldots,j,\ldots,n+1}\rangle_i = \langle\hat{\varepsilon}(x)_{1,\ldots,n}\rangle_i, \quad x\in v_i \ (i=1,\ldots,n) \ [1]).$$

For an ellipsoidal inclusion the field $\varepsilon(x)$ inside the inclusion is also uniform and

$$\langle\varepsilon(x)\rangle_i = A_i\langle\bar{\varepsilon}_i(x)\rangle, \; A_i = (I + P_iL_1)^{-1}, \tag{2.1}$$

where $P_i = (P_i^{\alpha\beta})\delta_{\alpha\beta}$ ($\alpha, \beta = 1,\ldots,N$), $P_i^{\alpha\alpha} = -\int U^{\alpha\alpha}(x-y)\; V_i(y)\; dy \; (x\in v_i)$ are constant tensors that do not depend on the physicomechanical properties and size (but not shape) of the ellipsoid v_i. The tensors $P_i^{\alpha\alpha}$ are expressed in terms of the known Eshelby tensor in the theory of elasticity [9] or its analog in transport problems [5].

For n inclusions in a field $\hat{\varepsilon}(x)_{1,\ldots,n}$ with allowance for (1.12), (2.1) and the EMF hypothesis we obtain ($j = 1,\ldots,n$, $j \neq i$)

$$\begin{gathered}\langle\bar{\varepsilon}(x)\,|\,x_1,\ldots,x_n\rangle_i - \sum_{j\neq i}^{n}(\bar{v}_i\bar{v}_j)^{-1}\int\int U(x-y)V_i(x)V_j(y)R_j\,\times\\ \times\langle\bar{\varepsilon}(y)\,|\,x_1,\ldots,x_n\rangle_j\,dx\,dy = \langle\hat{\varepsilon}(x)_{1,\ldots,n}\rangle, \; R_j = L_1^{(j)}A_j\bar{v}_j.\end{gathered} \tag{2.2}$$

To solve (2.2) by linear algebra methods we form the matrix T^{-1} with elements T^{-1}_{mk} ($m, k = 1,\ldots,n$) as the submatrix

$$\begin{gathered}T^{-1}_{mk} = I\delta_{mk} + (\delta_{mk}-1)R_mS(x_m-x_k),\\ S(x_m-x_k) = (\bar{v}_m\bar{v}_k)^{-1}\int\int U(x-y)V_m(x)V_k(y)\,dx\,dy,\end{gathered}$$

where

$$\langle\bar{\varepsilon}(x)\,|\,x_1,\ldots,x_n\rangle_i = R_i^{-1}\sum_{j=1}^{n}T_{ij}R_j\langle\hat{\varepsilon}(x)_{1,\ldots,n}\rangle_j. \tag{2.3}$$

Within the framework of the EMF with the aid of solutions (2.2), (2.3) we obtain from (1.11) a closed system of integral equations in the fields $\langle\hat{\varepsilon}(x)_{1,\ldots,j}\rangle_i$ ($j = 1,\ldots,n$, $i = 1,\ldots,j$):

$$\langle\hat{\varepsilon}(x)_{1,\ldots,j}\rangle_i = \langle\varepsilon\rangle + \int\Big\{S(x_i - x_q)\,\varphi(v_q\,|\,x_1, \ldots, x_j)\sum_{l=1}^{j+1} T_{ql}R_l\langle\hat{\varepsilon}(x)_{1,\ldots,j+1}\rangle_l - S_i(x_i - x_q)\langle R\hat{\varepsilon}_1\rangle\Big\}dx_q,$$
$$\langle\hat{\varepsilon}(x)_{1,\ldots,n}\rangle_i = \langle\varepsilon\rangle + \int\Big\{S(x_i - x_q)\,\varphi(v_q\,|\,x_1, \ldots, x_{n-1})\sum_{l=1}^{n} T_{ql}R_l\langle\hat{\varepsilon}(x_{1,\ldots,n}\rangle_l - S_i(x_i - x_q)\langle R\hat{\varepsilon}_1\rangle\Big\}dx_q \tag{2.4}$$

$\left(S_i(x_i - x_q) = \bar{v}_i^{-1}\int U(x - x_q)\,V_i(x)\,dx,\ x_q \in v_i\right)$. The solution of (2.4) can be effected numerically by estimating $\langle\hat{\varepsilon}(x)_{1,\ldots,n}\rangle_i$ ($i = 1,\ldots,n$) from the last of n rows of (2.4) by the method of successive approximations for all possible positions of $v_1,\ldots,v_n$. We substitute the value obtained for $\langle\hat{\varepsilon}(x)_{1,\ldots,n}\rangle_i$ ($i = 1,\ldots,n$) into the right side of the (n − 1)-th row of (2.4), etc., until we get the estimate $\langle\bar{\varepsilon}(x_i)\rangle_i \equiv \langle\bar{\varepsilon}(x_i)\rangle \equiv D_i\langle\varepsilon\rangle$ which makes it possible to determine $R^* = \sum_{i=1} \langle n_iR_iD_i\rangle$ and the tensor of effective properties (1.3) ($n_i = \langle V_i\rangle/\bar{v}_i$ is the calculated concentration of inclusions v_i).

The analytical solution of the problem can be found in the two-particle approximation and the assumption

$$\langle\hat{\varepsilon}(x)_{12}\rangle_i = \langle\bar{\varepsilon}(x_i)\rangle = \text{const}\ (i = 1, 2). \tag{2.5}$$

Then from (2.3), (2.5), and the first equation of (2.4) we obtain

$$\langle\bar{\varepsilon}(x_i)\rangle = \langle R_i\rangle_\omega^{-1}\sum_{j=1}^{N_c} n_i\left(Y^{-1}\right)_{ij}\langle R_j\rangle_\omega\langle\varepsilon\rangle, \tag{2.6}$$

where the tensor Y^{-1} takes the binary interaction of inclusions into account and has the reciprocal tensor Y, written in matrix form as the submatrices

$$\begin{aligned} Y_{ij} = \delta_{ij}\Big(I - \langle R_i\rangle_\omega\int\langle S(x_i - x_j)\rangle_\omega T_{ji}\varphi(v_j\,|\,x_j;\,x_i)\,dx_j\Big) + \\ + (\delta_{ij} - 1)\langle R_i\rangle_\omega\int\{\langle S(x_i - x_j)\rangle_\omega T_{ji}\varphi(v_j\,|\,x_j;\ x_i) - \langle S_i(x_i - x_j)\rangle_\omega n_j\}\,V(x_j; \\ x_i)\,dx_j - \langle R_i\rangle_\omega\langle P_i\rangle_\omega n_j,\ V(x_j;\,x_i) \equiv V(x_j) - V_i(x_j). \end{aligned} \tag{2.7}$$

In (2.6) and (2.7) for brevity of manipulations we have introduced the operator for averaging over possible orientations of the inclusions $v_i(\langle(\cdot)\rangle_\omega)$.

Equation (2.6) makes it possible to find expressions for the tensors of effective properties (1.3) of the concentrators B_α^* of field σ in the component α ($\langle\sigma\rangle_\alpha = B_\alpha^*\langle\sigma\rangle$, $\langle\sigma^\beta\rangle_\alpha = B_\alpha^{*\beta\gamma}\langle\sigma^\gamma\rangle$, $\alpha = 0, 1,\ldots,N_c$; $\beta,\ \gamma = 1,\ldots,N$)

$$L^* = L_0 + \sum_{i,j=1}^{N_c} n_i\left(Y^{-1}\right)_{ij}\langle R_j\rangle_\omega,$$
$$B_i^* = L^{(i)}A_i\sum_{j=1}^{N_c} n_i\left(Y^{-1}\right)_{ij}\langle R_j\rangle_\omega M^*\quad (i = 1, \ldots, N_c),\quad B_0^* = c_0^{-1}\Big(I - \sum_{i=1}^{N_c} c_iB_i^*\Big), \tag{2.8}$$
$$c_\alpha \equiv \langle V_\alpha\rangle.$$

3. Effective Strength Surface. By analogy with the problems of mechanical strength, we use the familiar method of constructing the effective strength surface $\Pi^*(\langle\sigma\rangle)$ [7] by substituting the mean values of the tensors into the components

$$\Pi^*(\langle\sigma\rangle) = \max_i\sum_\alpha\{\Pi_i^{1\alpha}B_i^*\langle\sigma\rangle + \Pi_i^{2\alpha}(B_i^*\otimes B_i^*)\langle\sigma\rangle\otimes\langle\sigma\rangle\} = 1. \tag{3.1}$$

Equations (3.1) give physically contradictory results. Indeed, for an isotropic medium the mechanical strength properties whose isotropic matrix satisfy the Mises criterion (N = 1), we obtain $\langle\sigma_{ij}\rangle_0 - \langle\sigma_{kk}\rangle\delta_{ij}/3 = 0$ under a hydrostatic load with any method of estimating B_0^*. This means that a porous material has an infinite strength under hydrostatic compression and this is a variance from experiment.

It is more exact to choose the effective strength surface in the form stemming from (1.2), (1.8) ($i = 0, 1,\ldots,N_c$):

$$\Pi^*(\langle\sigma\rangle) = \max_i\sum_{\alpha,\beta}\{\Pi_i^{1\alpha}B_i^{*\alpha\beta}\langle\sigma^\beta\rangle + \bar{v}_i^{-1}\Pi_i^{2\alpha}\partial M^*/\partial M^{(i)}(\langle\sigma\rangle\otimes\langle\sigma\rangle)\} = 1. \tag{3.2}$$

Here the tensors $B^{*\alpha\beta}$ and M^* are calculated from Eqs. (2.8). Thus, with the assumptions made the problem of constructing an effective strength surface is equivalent to the problem of evaluating the effective modulus. The mean values of the one-point first and second moments of the field σ in the components of the medium are used in Eqs. (3.2).

4. Examples. We consider the problem of calculating the effective mechanical strength of the components (N = 1). A specific expression for the modulus L^* of a porous material was obtained earlier [4] by taking the first terms of the series in the representation of the binary-interaction matrix T_{ij} into account. In particular, for an undistorted matrix with spherical pores of one size in the notation of [1] $(\mu^{(0)} = 2(L^{(0)}_{ijij} - L^{(0)}_{iijj}/3)/5,\ c \equiv \langle V\rangle)$ we have

$$M^* = ((\mu^{(0)})^{-1}c[2 - 29c/12]^{-1},\ \ 2(\mu^{(0)})^{-1}[1 + 5c\{3 - 35c/8\}^{-1}]). \tag{4.1}$$

Inclusion of the compressibility of the matrix in the parameter ranges $0.1 \leqslant c \leqslant 0.5$, $0.3 \leqslant \nu_0 \leqslant 0.5$ (ν_0 is Poisson's ratio) results in a refinement of M^* is no more than 5% in comparison with Eq. (4.1), which can be disregarded. Substituting (4.1) into (3.2) on the assumption that the strength properties satisfy the Mises strength criterion ($s_{ij}s_{ij} = k_0^2$, $s_{ij} = \sigma_{ij} - \sigma_{kk}\delta_{ij}/3$), we obtain an expression for the strength surface of the medium, written in terms of the invariants of the macrostress tensor:

$$\begin{gathered} I_2 + b^*I_1^2 = (k^*)^2,\ \ I_1 = \langle\sigma_{ii}\rangle,\ \ I_2 = \langle s_{ij}\rangle\langle s_{ij}\rangle, \\ b^* = c(2 - 35c/12)[(1 + 5c/24)(1 - 29c/24)]^{-1}, \\ (k^*)^2 = k_0^2(1 - c)(1 - 35c/24)[1 + 5c/24]^{-1}. \end{gathered} \tag{4.2}$$

We can show that for any pore concentration c the effective strength parameters b^* and k^* lie between the corresponding parameters of the known criteria of Garson [13] and Tverdgard [14].

When the pores are replaced by infinitely strong hard spherical inclusions with ideal adhesion to the matrix, we find the equation of the strength surface in the form (4.2) with the parameters

$$(k^*)^2 = k_0^2(1 - c)(1 + 9c/16)(1 - 31c/16)^{-1},\ \ b^* = 0, \tag{4.3}$$

i.e., the strength of a composite medium increases with the degree of filling. Relation (4.3) formally coincides with the effective yield stress of rigid-plastic composites [4].

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